Update on loop calculations in the Gribov-Zwanziger Lagrangian

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Outline

- Brief review of Gribov-Zwanziger Lagrangian in the Landau gauge
- Recent calculations at one and two loops
- For example, Gribov gap equation at two loops in the three dimensional theory
- One loop static potential in Gribov-Zwanziger Lagrangian
- Enhancement of bosonic Zwanziger localizing ghost and its effect on the static potential

Background to Landau gauge Gribov problem

- Yang-Mills action S is invariant under gauge transformations

$$A^a_\mu \;\to\; \tilde{A}^a_\mu \;=\; U^\dagger \partial_\mu U \;+\; U^\dagger A_\mu U$$

where

$$S \;=\; -\,\frac{1}{4} \int d^4x \, G^a_{\mu\nu} G^{a\,\mu\nu}$$

with $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

- In a non-abelian gauge theory there is a problem fixing the gauge globally [Gribov]
- For a given A^a_μ satisfying the gauge condition $\partial^\mu A^a_\mu = 0$ there are (Gribov) copies $\tilde{A}^a_\mu$ obeying the same condition $\partial^\mu \tilde{A}^a_\mu = 0$
- The existence of Gribov copies is equivalent to the Faddeev-Popov operator having zero eigenvalues

$$\partial^\mu D_\mu \Lambda^a \;=\; 0$$

- To avoid the copy problem the integration region of the path integral must be restricted to the first domain bounded by the Gribov horizon which contains the origin, $A^a_\mu = 0$, and denoted by Ω

Consequences

- Restriction to Ω via the no pole condition on $\mathcal{M}^{ab}(A) = (\partial^\mu D_\mu)^{ab}$ means a natural mass parameter, γ, emerges
- It is central to the infrared or non-perturbative behaviour of the theory
- γ is not independent and satisfies the one loop $\overline{\mbox{MS}}$ gap equation [Gribov]

$$1 ~=~ C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right] a ~+~ O(a^2)$$

  where $a = g^2/(16\pi^2)$
- Gluon propagator is suppressed and the ghost propagator behaves as $1/(p^2)^2$ as $p^2 \rightarrow 0$
- Gribov's effective Lagrangian is non-local [Gribov, Zwanziger]

$$L^\gamma ~=~ -~ \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} ~+~ \frac{C_A\gamma^4}{2} A^a_\mu \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} ~-~ \frac{dN_A\gamma^4}{2g^2} ~+~ \ldots$$

- γ is defined by the horizon (or no-pole) condition which is equivalent to the Gribov gap equation

$$\left\langle A^a_\mu(x) \frac{1}{\partial^\nu D_\nu} A^{a\,\mu}(x) \right\rangle ~=~ \frac{dN_A}{C_A g^2}$$

Gribov-Zwanziger Lagrangian

- With path integral restricted to Ω Zwanziger constructed a completely local Lagrangian L^Z
- Zwanziger's Lagrangian involved additional ghost fields $\{\phi^{ab}_\mu, \bar{\phi}^{ab}_\mu; \omega^{ab}_\mu, \bar{\omega}^{ab}_\mu\}$
- These localize the non-locality

$$
\begin{aligned}
L^Z &= -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D_\mu c^a + i \bar{\psi}^{iI} \not{D} \psi^{iI} \\
&\quad + \bar{\phi}^{ab\,\mu} \partial^\nu \left(D_\nu \phi_\mu \right)^{ab} - \bar{\omega}^{ab\,\mu} \partial^\nu \left(D_\nu \omega_\mu \right)^{ab} \\
&\quad - g f^{abc} \partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c \right)^b \phi^{ec\,\mu} \\
&\quad - \frac{\gamma^2}{\sqrt{2}} \left(f^{abc} A^{a\,\mu} \phi^{bc}_\mu - f^{abc} A^{a\,\mu} \bar{\phi}^{bc}_\mu \right) - \frac{d N_A \gamma^4}{2 g^2}
\end{aligned}
$$

- Fields ω^{ab}_μ and $\bar{\omega}^{ab}_\mu$ are anti-commuting
- Lagrangian is also *renormalizable*, [Zwanziger; Schaden, Maggiore; Sorella et al], so that it can be used to perform calculations

- Change to real fields

$$\phi^{ab}_\mu = \frac{1}{\sqrt{2}} \left(\rho^{ab}_\mu + i\xi^{ab}_\mu \right) \ , \ \bar{\phi}^{ab}_\mu = \frac{1}{\sqrt{2}} \left(\rho^{ab}_\mu - i\xi^{ab}_\mu \right)$$

- Lagrangian becomes

$$\begin{aligned} L^{GZ} &= L^{QCD} + \frac{1}{2}\rho^{ab\,\mu}\partial^\nu \left(D_\nu \rho_\mu\right)^{ab} + \frac{i}{2}\rho^{ab\,\mu}\partial^\nu \left(D_\nu \xi_\mu\right)^{ab} \\ &- \frac{i}{2}\xi^{ab\,\mu}\partial^\nu \left(D_\nu \rho_\mu\right)^{ab} + \frac{1}{2}\xi^{ab\,\mu}\partial^\nu \left(D_\nu \xi_\mu\right)^{ab} \\ &- \bar{\omega}^{ab\,\mu}\partial^\nu \left(D_\nu \omega_\mu\right)^{ab} - \frac{1}{\sqrt{2}} g f^{abc}\partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c\right)^b \rho^{ec\,\mu} \\ &- \frac{i}{\sqrt{2}} g f^{abc}\partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c\right)^b \xi^{ec\,\mu} - i\gamma^2 f^{abc} A^{a\,\mu} \xi^{bc}_\mu - \frac{dN_A \gamma^4}{2g^2} \end{aligned}$$

- Propagators are

$$
\begin{aligned}
\langle A^a_\mu(p) A^b_\nu(-p) \rangle &= -\frac{\delta^{ab} p^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \xi^{bc}_\nu(-p) \rangle &= \frac{i f^{abc} \gamma^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \rho^{bc}_\nu(-p) \rangle &= 0 \\
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle &= -\frac{\delta^{ac} \delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2 [(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle \xi^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &= 0 \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &= \langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle = -\frac{\delta^{ac} \delta^{bd}}{p^2} \eta_{\mu\nu}
\end{aligned}
$$

  where $P_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$
- There is mixing in the $\{A^a_\mu, \xi^{ab}_\mu\}$ sector
- The presence of the non-zero γ leads to a gluon propagator which is suppressed in the infrared

Updated results

- Horizon condition equates to

$$f^{abc}\langle A^{a\,\mu}(x)\xi^{bc}_{\mu}(x)\rangle \;=\; \frac{idN_A\gamma^2}{g^2}$$

which defines γ through the gap equation from the equation of motion

$$\xi^{ab}_{\mu} \;=\; i\gamma^2 f^{abc}\frac{1}{\partial^{\nu}D_{\nu}}A^{c}_{\mu}$$

- Renormalization group invariant coupling constant derived from the gluon and Faddeev-Popov ghost propagator form factors freezes in infrared to

$$\alpha^{\mathrm{eff}}_{s}(0) \;=\; \frac{16}{\pi C_A}$$

- Have checked that at one loop gluon propagator remains suppressed and transverse
- Kugo-Ojima confinement criterion holds at one loop ($\overline{\mathrm{MS}}$ and MOM schemes)

- Kugo and Ojima derived a necessary condition for confinement which was established using BRST symmetry and cohomology arguments
- Write full ghost propagator in the form

$$G_c(p^2) \ = \ \frac{1}{p^2[1+u(p^2)]} \ \equiv \ \frac{D_c(p^2)}{p^2}$$

- Kugo-Ojima confinement condition requires ghost enhancement in the infrared which corresponds to $1/(p^2)^2$ behaviour as $p^2 \rightarrow 0$ which is equivalent to $u(0) = -1$
- Can compute the ghost 2-point function in the Gribov-Zwanziger Lagrangian and expand to $O(p^2)$ at two loops
- The Kugo-Ojima condition is satisfied at two loops provided the Gribov mass gap condition is used
- Similar enhancement for ω^{ab}_μ at two loops
- Decoupling solution has no gluon suppression and no ghost enhancement

- Reproduce one loop gap equation by integrating mixed propagator using dimensional regularization in $\overline{\text{MS}}$ scheme
- Two loop $\overline{\text{MS}}$ correction to gap equation (massless quarks) with $s_2 = (2\sqrt{3}/9)\text{Cl}_2(2\pi/3)$

$$
\begin{aligned}
1 &= C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a \\
&+ \left[C_A^2 \left(\frac{3893}{1536} - \frac{22275}{4096} s_2 + \frac{29}{128} \zeta(2) - \frac{65}{48} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right.\right. \\
&\qquad \left. + \frac{35}{128} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 + \frac{411}{1024} \sqrt{5} \zeta(2) - \frac{1317\pi^2}{4096} \right) \\
&\quad + C_A T_F N_f \left(- \frac{25}{24} - \zeta(2) + \frac{7}{12} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right. \\
&\quad \left.\left. - \frac{1}{8} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 + \frac{\pi^2}{8} \right) \right] a^2 + O(a^3)
\end{aligned}
$$

- Two loop $\overline{\text{MS}}$ gap equation with massive quarks is also available [Ford & Gracey]
- Can invert $\overline{\text{MS}}$ gap equation to find a γ as a non-perturbative function of the coupling constant

Static potential

- One property of QCD is that gluons and quarks are confined and the potential between a pair of such fields rises linearly
- Can compute the potential between coloured sources using the Wilson loop either on the lattice or in perturbation theory
- For the latter this is known to three loops in $\overline{\mathrm{MS}}$ [Anzai, Kiyo, & Sumino; Smirnov, Smirnov & Steinhauser]
- It involves a different type of propagator similar to that used in heavy quark effective theory
- Can extend the perturbative static potential in QCD to that for the Gribov-Zwanziger Lagrangian
- One loop calculation is complete

Definitions

- For the static potential the Wilson loop is defined to be a rectangle of spatial length r and temporal length t
- Due to non-abelian character of QCD the potential requires a path ordering, giving the coordinate space form

$$V(r) = - \lim_{t \to \infty} \frac{1}{it} \ln \left\langle 0 \left| \mathrm{Tr} \mathcal{P} \exp \left(ig \oint dx^\mu T^a A^a_\mu \right) \right| 0 \right\rangle$$

- Place two static sources at $\pm \frac{1}{2} \mathbf{r}$ then this is equivalent to a partition function with the source term $J^\mu A^a_\mu$ where

$$J^a_\mu(x) = g v_\mu T^a \left[\delta^{(3)} \left(\mathbf{x} + \frac{1}{2} \mathbf{r} \right) - \delta^{(3)} \left(\mathbf{x} - \frac{1}{2} \mathbf{r} \right) \right]$$

- The vector v_μ is defined as $v_\mu = \delta_{\mu 0}$
- Potential can be computed in momentum space and Fourier transformed to coordinate space

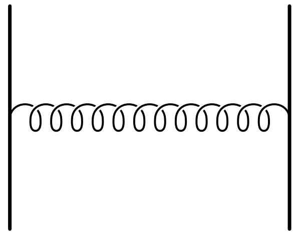
$$V(\mathbf{r}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i \mathbf{k}.\mathbf{r}} V(\mathbf{k})$$

- Performing the angular integrations this equates to

$$V(r) \;=\; \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 V(k)\frac{\sin(kr)}{kr}$$

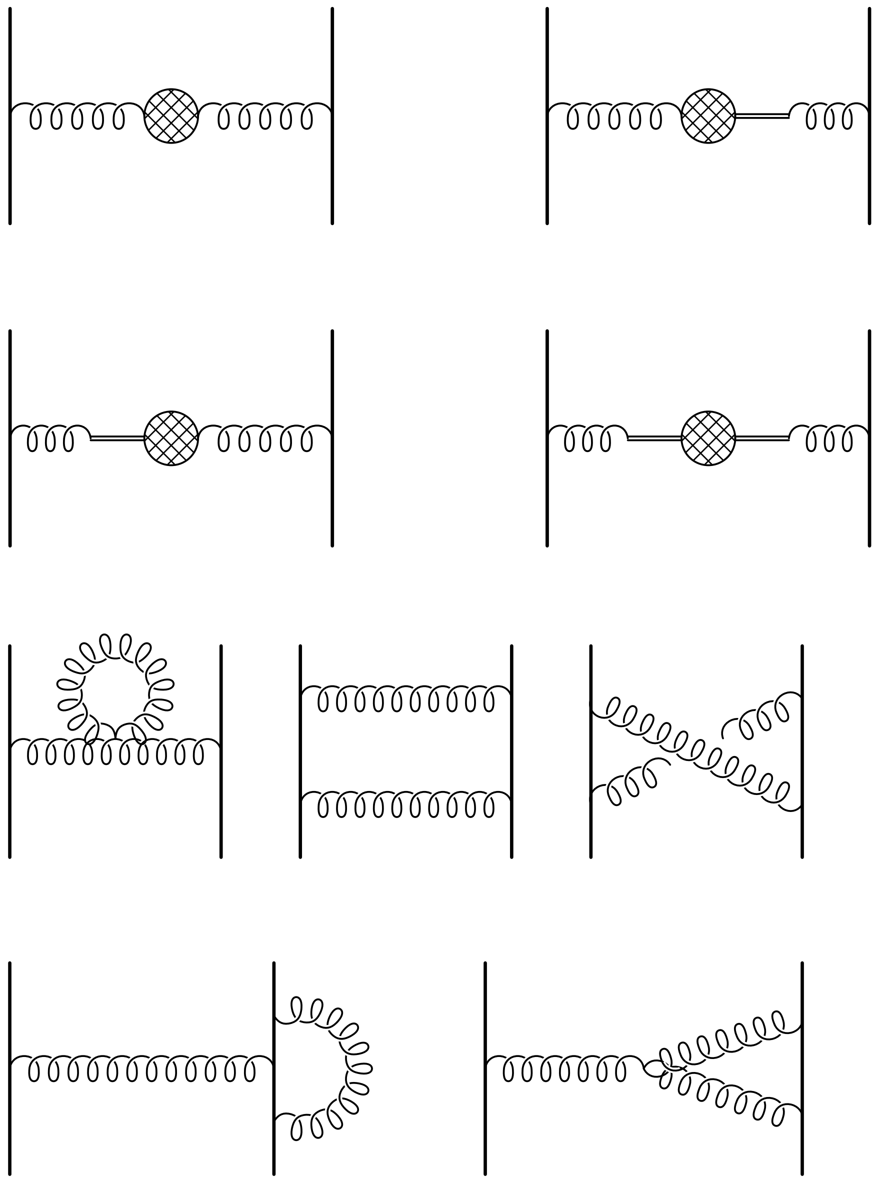
- The presence of the (heavy) static colour sources modifies the Feynman rules
- The source acquires a propagator $\frac{i}{pv}$ where p is the momentum
- The gluon-source coupling is $igT^a v_\mu$ with all other Feynman rules unaltered
- Previously the canonical QCD Lagrangian has been considered. Now replace with the Gribov-Zwanziger Lagrangian
- The localizing Zwanziger ghost fields are regarded as completely internal and do not couple directly to the sources
- The static potential is a gauge independent object
- In perturbation theory the non-Gribov case has been computed in Feynman gauge originally [Susskind, Fischler, Peter] at various loops
- At two loops the arbitrary gauge result has been determined by Schröder

- As Gribov-Zwanziger is specifically Landau gauge the expression which emerges must agree with arbitary gauge result as $\gamma \to 0$
- There are 31 one loop graphs generated using QGRAF
- Computed using code written in symbolic manipulation language FORM
- At leading order there is only a simple exchange



- At one loop there are propagator and vertex source corrections
- Full one loop expression is long

One loop graphs



- For instance it has terms of the form

$$
\begin{aligned}
V(\mathbf{p}) \quad = \quad & - \frac{C_F \mathbf{p}^2 g^2}{[(\mathbf{p}^2)^2 + C_A \gamma^4]} \\
& - \left[\frac{\sqrt{2}}{\gamma^2} \left[\frac{\sqrt{C_A}}{768} \ln \left[\frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2} \right] \sqrt{-1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2}}} \right. \right. \\
& \qquad - \frac{\sqrt{C_A}}{768} \ln \left[1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2}} \right] \sqrt{-1 + \sqrt{1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2}}} \\
& \qquad \left. \left. - \frac{\sqrt{C_A}}{48 \sqrt{2}} \tan^{-1} \left[\frac{\sqrt{C_A} \gamma^2}{\mathbf{p}^2} \right] + \ldots \right] + \ldots \right] \frac{C_F g^4}{16 \pi^2} \; + \; O(g^6)
\end{aligned}
$$

- Full expression agrees with usual perturbative expression

$$
\begin{aligned}
\lim_{\gamma \to 0} V(\mathbf{p}) \quad = \quad & - \frac{4 \pi C_F \alpha_s(\mu)}{\mathbf{p}^2} \left[1 \; + \; \left[\left[\frac{31}{9} - \frac{11}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] \right] C_A \right. \right. \\
& \left. \left. + \left[\frac{4}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] - \frac{20}{9} \right] T_F N_f \right] a \; + \; O(a^2) \right]
\end{aligned}
$$

- One loop potential includes the following terms

$$V(\mathbf{p}) \;=\; + \left[\frac{\pi C_A^{3/2} \gamma^2}{384 (\mathbf{p}^2)^2} \;-\; \frac{C_A^{3/2} \gamma^2}{192 (\mathbf{p}^2)^2} \tan^{-1} \left[\frac{\sqrt{C_A} \gamma^2}{\mathbf{p}^2} \right] \right] \frac{C_F g^4}{16 \pi^2} \;+\; \ldots$$

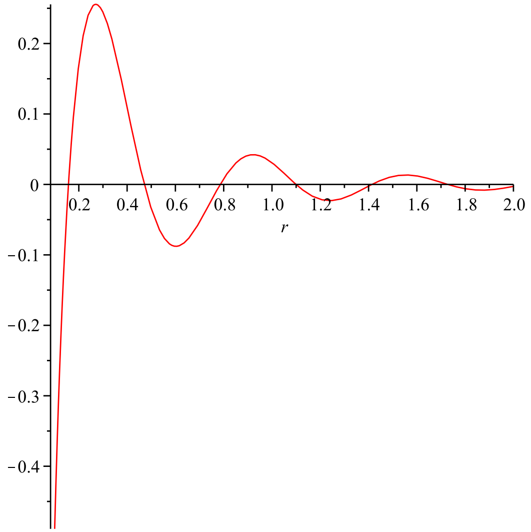
- Fourier transform of the first term leads to linear potential

$$V(r) \;=\; - \frac{C_F C_A^{3/2} \gamma^2 g^4}{49152 \pi^2} r \;+\; \ldots$$

- However confining behaviour as $r \to 0$ is cancelled by Fourier transform of second term if $\gamma^2 > 0$
- Potential has threshold effect at $p^2 = 2 \sqrt{C_A} \gamma^2$
- Similar point to that found by Zwanziger in the physical cut of the spectral density representation of the field strength correlation function at leading order
- Zwanziger suggested it was a potential candidate for a glueball mass

- Leading order part of potential leads to a Friedel potential due to non-zero width

$$V(r) \;=\; -\,\frac{C_F g^2}{4\pi r}\,\exp\left[-\frac{C_A^{1/4}\gamma r}{\sqrt{2}}\right]\cos\left(\frac{C_A^{1/4}\gamma r}{\sqrt{2}}\right) \;+\; O(g^4)$$



- Yukawa potential for a purely massive field is always negative
- One loop potential in coordinate space should be similar
- Seems higher orders in perturbation theory will match the rising potential accessed by a non-perturbative route

- Next define the V-scheme coupling constant via

$$V(\mathbf{p}) = - \frac{4\pi C_F \alpha_V(\mathbf{p})}{\mathbf{p}^2}$$

- Since static potential is constructed from gauge invariant Wilson loop, this could be regarded as a gauge independent definition of the strong coupling constant
- At one loop the Gribov-Zwanziger static potential result would imply $\alpha_V(0) = 0$ since

$$\begin{aligned}\tilde{V}(\mathbf{p}) &= - \frac{C_F \mathbf{p}^2 g^2}{C_A \gamma^4} - C_F \left[\frac{\pi\sqrt{C_A}}{32\gamma^2} + \left(\frac{13}{72} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right) \frac{\mathbf{p}^2}{\gamma^4} \right] \frac{g^4}{16\pi^2} \\ &+ O((\mathbf{p}^2)^2; g^6)\end{aligned}$$

Three dimensions

- Have repeated calculations for three dimensional Gribov-Zwanziger Lagrangian
- Theory is ultraviolet finite but has same features as four dimensions such as enhancement and Kugo-Ojima criterion is satisfied at two loops
- Gap equation has simple form (coupling constant has a dimension)
- For $N_f = 0$

$$\frac{3}{4} = \frac{\sqrt{2} C_A^{3/4} g^2}{16\pi\gamma} + \left[\frac{917\pi}{262144} + \frac{17}{98304} + \frac{545}{131072} \tan^{-1} \left[\frac{3}{4} \right] \right] \frac{C_A^{3/2} g^4}{\pi^2 \gamma^2} + O(g^6)$$

- Solving for $x_n = g^2/(4\pi C_A^{1/4}\gamma)$ at nth loop find $x_1 = 0.7071$ and $x_2 = 0.4206$ for $SU(3)$
- For non-zero γ static potential has no linearly rising term in contrast with $\gamma = 0$ case
- Effective renormalization group invariant coupling constant also freezes to a non-zero value

$$\alpha_s^{\mathrm{eff}}(0) = \frac{3\sqrt{2}}{4} C_A^{1/4} \gamma$$

Bosonic ghost enhancement

- The fact that the Kugo-Ojima criterion is satisfied suggests that the Gribov-Zwanziger Lagrangian describes a confined gluon since the Faddeev-Popov ghost is enhanced
- One loop static potential does not have a dipole due to the absence of the exchange of a single enhanced spin-1 coloured object
- Faddeev-Popov ghost is Grassmann and thus is excluded as an exchange particle
- Recently Zwanziger has analysed the ξ^{ab}_μ propagator using a Dyson Schwinger approach and shown that ξ^{ab}_μ overenhances
- Possible to see *enhancement* in the perturbative set-up
- ρ^{ab}_μ enhances at two loops similar to c^a and ω^{ab}_μ
- Requires study of the mixing matrix of 2-point functions in the $\{A^a_\mu, \xi^{ab}_\mu\}$ sector
- Clue in the ξ^{ab}_μ sector since the Lagrangian kinetic term colour sector enhances
- Write matrix of 2-point functions in $\{A^a_\mu, \xi^{ab}_\mu\}$ formally as

$$\Lambda_2^{\{ab|cd\}} = \begin{pmatrix} \mathcal{X} \delta^{ac} & \mathcal{U} f^{acd} \\ \mathcal{U} f^{cab} & \mathcal{Q}_\xi^{abcd} \end{pmatrix}$$

- General decomposition into colour tensors

$$\mathcal{Q}^{abcd}_\xi = \mathcal{Q}_\xi \delta^{ac}\delta^{bd} + \mathcal{W}_\xi f^{ace} f^{bde} + \mathcal{R}_\xi f^{abe} f^{cde} + \mathcal{S}_\xi d_A^{abcd} + \mathcal{P}_\xi \delta^{ab}\delta^{cd} + \mathcal{T}_\xi \delta^{ad}\delta^{bc}$$

where $d_A^{abcd} = \frac{1}{6}\mathrm{Tr}\left(T^a_A T^{(b}_A T^c_A T^{d)}_A\right)$

- Transverse part of 2-point function in the zero momentum limit in $\overline{\mathrm{MS}}$ is

$$\begin{aligned}\langle \xi^{ab}_\mu(-p)\bar{\xi}^{cd}_\nu(p)\rangle^{-1} = & \left[\delta^{ac}\delta^{bd}\left[1 - C_A\left(\frac{5}{8} - \frac{3}{8}\ln\left(\frac{C_A\gamma^4}{\mu^4}\right)\right)a\right]p^2 \right. \\ & + \frac{7}{144} f^{ace} f^{bde} p^2 a + \frac{11}{288} f^{abe} f^{cde} p^2 a \\ & \left. + \frac{7}{24} d_A^{abcd} \frac{p^2}{C_A} a + O(a^2)\right] P_{\mu\nu}(p) \\ & + O\left((p^2)^2\right)\end{aligned}$$

- The key piece is $\mathcal{Q}_\xi$ which is effectively the gap equation

- Inverting the full mixing matrix requires the solution of nine algebraic equations
- Have checked the equations reproduce the original propagators and the one loop corrections
- Satisfying gap equation prior to inversion, similar to the Faddeev-Popov ghost case, produces enhanced (transverse) ξ^{ab}_μ and ρ^{ab}_μ propagators in the infrared limit
- Leading terms in $p^2 \to 0$ limit are

$$
\begin{aligned}
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle &\sim -\frac{4\gamma^2}{\pi\sqrt{C_A}(p^2)^2 a} \left[\delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} - \frac{2}{C_A} f^{abe} f^{cde} \right] P_{\mu\nu}(p) \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &\sim -\frac{8\gamma^2}{\pi\sqrt{C_A}(p^2)^2 a} \delta^{ac}\delta^{bd} P_{\mu\nu}(p)
\end{aligned}
$$

- Colour channel of the enhanced part does not correspond to the ξ^{ab}_μ kinetic term unlike ρ^{ab}_μ whose enhancement is similar to Faddeev-Popov and anti-commuting localizing ghosts
- Enhancement of ρ^{ab}_μ extends to two loops, similar to ω^{ab}_μ

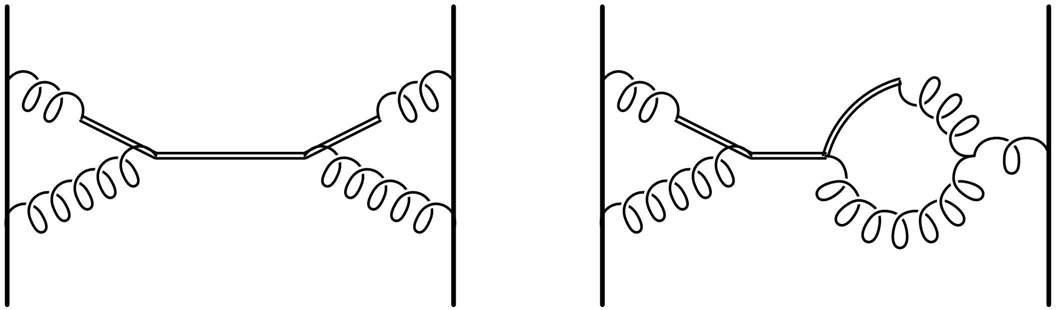
- Do not observe over-enhancement in this one loop analysis
- Contracting ξ^{ab}_μ with f^{abc} leads to a non-enhanced propagator for this combination
- In three dimensions the colour structure of the enhanced part of the ξ^{ab}_μ propagator is the same
- More generally taking the Laurent expansion of the propagator in powers of $\mathcal{Q}_\xi$ gives

$$\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle \sim \frac{1}{2\mathcal{Q}_\xi} \left[\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} - \frac{2}{C_A} f^{abe} f^{cde} \right] P_{\mu\nu}(p)$$

- $\mathcal{Q}_\xi$ is $O\big((p^2)^2\big)$ when gap equation is implemented

Enhancement and static potential

- Can examine the consequences of an enhanced bosonic spin-1 adjoint field in the static potential formalism
- However ξ^{ab}_μ or ρ^{ab}_μ do not directly couple to coloured sources of formalism
- Indirect single exchange of ξ^{ab}_μ via graphs of the form



- Second graph is a self-energy correction to mixed propagator
- First graph is indirect exchange of ξ^{ab}_μ
- Examine the eight contributing Feynman diagrams in the $p^2 \to 0$ limit, with sources in the adjoint

- First with the usual ξ^{ab}_μ propagator in the $p^2 \to 0$ limit

$$\tilde{V}^{2\,\mathrm{loop}}(\mathbf{p}) \;=\; \left[\frac{5\pi C_A^{3/2} (\mathbf{p}^2)^2}{4608\gamma^6} \;+\; O\left((\mathbf{p}^2)^2 \right) \right] \frac{g^6}{(16\pi^2)^2}$$

which vanishes in zero momentum limit

- Replace ξ^{ab}_μ propagator with enhanced version to find, as $p^2 \to 0$,

$$\tilde{V}^{2\,\mathrm{loop}}(\mathbf{p}) \Big|^{SU(3)}_{\mathrm{enhanced}} \;=\; - \left[\frac{3\mathbf{p}^2}{14\gamma^4} \;+\; O\left((\mathbf{p}^2)^2 \right) \right] \frac{g^4}{16\pi^2}$$

- The enhancement drops the original momentum dependence by *one* power and does not produce a dipole
- Colour structure excludes further reduction in power
- There is a reordering of perturbation theory
- In original Gribov formulation this graph involves an effective four gluon vertex

Speculation

- It seems perturbation theory does not produce linearly rising potential despite Kugo-Ojima criterion being satisfied at two loops
- There may be several possibilities to access this
- First one may need to carry out a strong coupling expansion [Zwanziger] which will match onto perturbative result beyond its range of validity
- Or one has to modify the original Lagrangian with, say, additional non-local operators which do not upset Kugo-Ojima criterion but maybe lose renormalizability though retaining a degree of renormalizability order-by-order in perturbation theory
- For example

$$g\gamma^4 f^{abc} G^a_{\mu\nu} \frac{1}{\partial^\sigma D_\sigma} A^{b\,\mu} \frac{1}{\partial^\rho D_\rho} A^{c\,\nu}$$

- A similar expansion appears for the non-local operator

$$\min_{\{U\}} \int d^4x \, (A^{a\,U}_\mu)^2$$

- Or revisit some of the assumptions in the original Gribov construction
- Or something else

Conclusions

- One loop correction to static potential in Gribov-Zwanziger Lagrangian does not produce a dipole term
- Shape of potential does allow, in principle, for smooth matching onto linearly rising piece in both three and four dimensions
- Enhancement (but not overenhancement) of ξ^{ab}_μ and ρ^{ab}_μ can be seen in perturbative set-up but does not produce dipole due to subtleties of group structure
- Its effect in Schwinger-Dyson equations needs to be examined in detail
- Overenhancement would not appear to improve situation in perturbation theory
- Other colour channels of bosonic ghosts may enhance which requires two loop calculation in zero momentum limit (> 1000 Feynman graphs)