Gluon-Ghost Formulation of QCD –

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- New functional approach to non-perturbative QCD.

General Properties:

1) Define Schwingerian functional rep. for $\mathcal{Z}_{QCD} = \langle 0 | \left(e^{i\int [j\cdot A + \bar{\eta}\psi + \bar{\psi}\eta]} \right)_+ | 0 \rangle$, which is MGI and MLC; impossible for QED but uniquely possible for QCD.

- Effect is to treat all gluon exchanges between Qs and/or $\bar{Q}$s as "virtual ghost particles", with $D_{c,\mu\nu}^{(\varepsilon)ab}$ removed;
  ∴ MGInv. by MGInd.

2) Convenient re-arrangement of Sch. sol'ns for all n-pt. QCD fnct'ns in terms of "linkage operations" on $\mathcal{F}[G_c[A], L[A]]$, where:

$$G_c[A] = [m + \gamma\cdot\partial - ig\gamma\cdot A\cdot\lambda]^{-1},\ L[A] = \mathrm{Tr}\ln[1 - ig\gamma\cdot A\cdot\lambda\, S_c],\ S_c = G_c[0].$$

- Definition of "Linkage operator": $\mathcal{D}_A = -\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}$,

$e^{\mathcal{D}_A}\mathcal{F}[A]$ connects all A-dep of $\mathcal{F}[A]$ in pairwise fashion.

3) Fradkin functional reps. for $G_c[A]$ and $L[A]$, where both $G_c[A]$ and $L[A]$ are given as F.I.s over exponentials of linear and quadratic A-dep.

∴ All linkage operations can be performed exactly; what remains are the Fradkin F.I.s which can be easily approximated in different physical situations.

In simplest, quenched, eikonal example below, Fradkin reps not needed; but importance of the method is that, in general, Σ all Feynman graphs can be included.

4) <u>Results take a simple and important form of "Effective Locality"</u>, as in eikonal calculation; but EL is true in general.
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5) <u>Idealistic vs. Realistic QCD</u>

A) All quanta of all fields are measurable, with asymptotic states defined in terms of q. nos. ← Ideal QFT.

B) Realistic QCD: Color is confined; all asymp. q-nos. cannot be defined (e.g., $p_\perp$ of conf./bound $Q/\bar{Q}$).

∴ We begin QCD analysis using A, but find - as a result of EL that transition to B is unavoidable.

<u>→ All details in above Reference.</u>

6) <u>Sch. funct. sol'n in QED (rearranged):</u>

$$\mathfrak{Z}_{QED}[j, \eta, \bar{\eta}] = N e^{\frac{i}{2}\int j D_c j} \cdot e^{\mathcal{D}_A} e^{i\int \bar{\eta} G_c[A] \eta + L[A]} \Big|_{A = \int D_c j} ,$$

$$N^{-1} = \langle 0 | S | 0 \rangle = e^{\mathcal{D}_A} e^{L[A]} \Big|_{A \to 0} .$$

<u>NB</u>: $G_c[A]$, $L[A]$ ⇐ Pot. theory; $e^{\mathcal{D}_A} \dots$ ⇒ QFTh.

$$\mathcal{Z}_{QCD}[J,\eta,\bar\eta] = N' e^{\frac{i}{2}\int J D_c J} \cdot e^{\mathfrak{D}_A} e^{-\frac{i}{4}\int F^2 + \frac{i}{2}\int A (D_c)^{-1} A} \cdot e^{i\int \bar\eta G_c[A]\eta + L[A]} \Big|_{A_\mu^a = \int D_{c\,\mu\nu}^{ab} J_\nu^b} .$$

with : $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$.

Initial, gauge-dep. $e^{\frac{i}{2}\int J D_c^{(0)} J}$ not relevant to $Q/\bar Q$ processes, and is dropped.

Our example of $Q/\bar Q$ scattering requires :

$$M \sim e^{\mathfrak{D}_A} G_c^{I}[A] G_c^{II}[A] \cdot e^{L[A] + \frac{i}{4}\int F^2} \cdot e^{\frac{i}{2}\int A (D_c)^{-1} A} \Big|_{A\to 0} .$$

· Use Halpern rep. ('70s) : $e^{-\frac{i}{4}\int F^2} = N'' \int d[\chi] e^{\frac{i}{4}\int \chi^2 + \frac{i}{2}\int \chi\cdot F}$.

· In quenched approx, $L[A] \to 0$, $N' \to 1$.

· In eikonal approx., well-defined in QCD, as in Abelian theories,

$$T(s,t) = \frac{is}{2M^2}\int d^2b\, e^{i\vec q\cdot\vec b}\left[1 - e^{i\chi(s,b)}\right] , \quad s = -(p_1+p_2)^2$$
$$t = -(p_1 - p_1')^2 = -q^2 \xrightarrow{CM} -\vec q^{\,2} .$$

· In this Eikonal calc, neglect all $Q/\bar Q$ self-energy structure; retain only interactions between Q_I and $Q_{II}/\bar Q_{II}$.

In Abelian theory : $G_c(x,y|A) \sim e^{ig p_\mu \int_{-\infty}^{+\infty} ds\, A_\mu(y - sp)}$.

In QCD : $G_c(x,y|A) \sim \left(e^{ig p_\mu \int_{-\infty}^{+\infty} ds\, A_\mu^a(y-sp)\lambda^a}\right)_+$

$$\to \int d[\alpha]\, \delta[\alpha^a(s) - g p_\mu A_\mu^a(y-sp)] \left(e^{i\int \alpha\cdot\lambda}\right)_+$$

$$\rightarrow \int d[\alpha] \cdot N'' \int d[\Omega]\, e^{i\int_{-\infty}^{\infty} \Omega_a(s)\left[\alpha^a(s) - g\, p_\mu A^a_\mu(y-sp)\right]} \left(e^{i\int \alpha\cdot\lambda}\right)_+$$

$$\Rightarrow N'' \int d[\alpha] \int d[\Omega]\, e^{i\int \Omega\cdot\alpha} \left(e^{i\int \alpha\cdot\lambda}\right)_+ e^{-i\int d^4w\, A^a_\mu(w) Q^a_\mu(w)} \,,$$

$$\text{with}: \quad Q^a_\mu(w) = g\, p_\mu \int_{-\infty}^{+\infty} ds\, \Omega_a(s)\, \delta(w - y + sp) \,.$$

· Subsequently, because of EL, $\int d[\alpha] \int d[\Omega] \Rightarrow \int d^n\alpha(0) \cdot \int d^n\Omega(0)$.

NB : Factors of $\exp\left[ig\int \Omega^a_{\mu\nu} F^a_{\mu\nu}(x-sp)\right] \Rightarrow 1$, in eikonal models of QED and QCD.

→ What remains to be calculated :

$$\cancel{e^{i\chi}} \sim \int d[\chi]\, e^{\frac{i}{4}\int \chi^2} \int d[\alpha_I] \int d[\Omega_I] \cdots \int d[\alpha_{II}] \int d[\Omega_{II}] \cdots$$

$$e^{\mathcal{D}_A} \cdot e^{\frac{i}{2}\int A\cdot K\cdot A + i\int A\cdot(Q_I + Q_{II} - \partial\chi)} \Big|_{A\to 0} \,,$$

where :

$$Q^a_\mu(w) = g\, p_\mu \int_{-\infty} ds\, \Omega_a(s)\, \delta(w - y + sp) \,,$$

$$K^{ab}_{\mu\nu} = g\, f_{abc}\, \chi^c_{\mu\nu} + (D_c^{-1})^{ab}_{\mu\nu} \,,$$

and:

$$\langle w | \chi^c_{\mu\nu} | w' \rangle = \delta(w - w')\, \chi^c_{\mu\nu}(w) \,, \quad \langle w | D^{ab}_{G\mu\nu} | w' \rangle = D^{ab}_{G\mu\nu}(w - w') \,.$$

Here is where the "ghost magic" appears :

$$e^{-\frac{i}{2}\int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}} \cdot e^{\frac{i}{2}\int AKA + i\int A\cdot(\underbrace{Q_I + Q_{II} - \partial\chi}_{Q})} \Big|_{A\to 0}$$

$$= e^{-\frac{1}{2}\mathrm{Tr}\ln(1 - KD_c)} \cdot e^{\frac{i}{2}\int Q\cdot\left[D_c \frac{1}{1 - KD_c}\right]\cdot Q} \,,$$

where : $K = (D_c)^{-1} + g f\cdot\chi$.

Then: $1 - K D_c = 1 - [(D_c)^{-1} + g f\cdot\chi] D_c \Rightarrow -(g f\cdot\chi)^{-1} D_c$,

$\therefore \hookrightarrow e^{-\frac{1}{2}\mathrm{Tr}\ln(-g f\cdot\chi D_c)} \cdot e^{-\frac{i}{2g}\int Q\cdot D_c\cdot[(f\cdot\chi)\cdot D_c]^{-1}\cdot Q}$

Q-dep: $-\frac{i}{2g}\int Q\cdot D_c\cdot(D_c)^{-1}(f\cdot\chi)^{-1}\cdot Q = -\frac{i}{2g}\int Q\cdot(f\chi)^{-1}\cdot Q$,

and all D_c-dep. has been removed from the interaction!

(The $\ln(D_c)$ can be absorbed into the normalization.)

Q-dep.: $-\frac{i}{2g}\iint Q(w)\,[f\cdot\chi(w)]^{-1}\delta(w-w')\,Q(w') = -\frac{i}{2g}\int Q(w)\,(f\cdot\chi(w))^{-1} Q(w)$

- For int. between I and II, this becomes:

$-\frac{i}{2g}(g^2)\, P_{1\mu} P_{2\nu}\int d^4w\,\delta(w-y_1+s_1p_1)\,\delta(w-y_2+s_2p_2)\,(f\cdot\chi(w))^{-1}\Big|_{\mu\nu}^{ab} \Omega_a^{I}(s_1)\,\Omega_b^{II}(s_2)$

$\quad -\frac{i}{2g}\int d^4w\,(\partial_\lambda \chi^a_{\mu\lambda}(w))\cdot(f\cdot\chi(w))^{-1}\cdot(\partial_\sigma \chi^b_{\nu\sigma}(w))$.

for $g \gg 1$, drop the $(\partial\chi)\cdot(f\chi)^{-1}\cdot(\partial\chi)$ term,
and retain the $O(g)\; P_{1\mu}P_{2\nu}\cdots$ term.

- For HE scattering, $\delta\cdot\delta$ leads to: $i g\,\delta^{(2)}(\vec{b})\,\Omega^a_I(0)\,\Omega^b_{II}(0)\,(f\cdot\chi(w$

where: $w^0_\mu = (\vec{y}_{1\perp}, 0_L; y_0)$, $\vec{b} = (\vec{y}_1 - \vec{y}_2)_T$.

$\therefore$ Only $w_\mu \to w^{(0)}_\mu$ of the Halpern FI is relevant; all other
values are removed (by their normalization factors).
Also, $\Omega_{I,II}(s_{1,2}) \to \Omega_{I,II}(0)$, corresponding to color changes
occurring at distance of closest approach.

Using "Realistic" QCD, as in (58), $\delta^{(2)}(\vec{b}) \to \varphi(mb)$,
since
where $\varphi(mb) \to (2\pi)^{-2}\int d^2k_\perp\, e^{i\vec{k}_\perp\cdot\vec{b}} \cdot e^{-(k_\perp^2/m^2)}$, as simple example,

since large $p_\perp$ of bound $q/\bar{q}$ cannot be measured; and all one knows is that, for a bound state, arb. large $k_\perp$ cannot be included.

For this example, $\varphi(mb) \sim m^2 e^{-b^2m^2/4}$,

and in Eik. model: $m \sim O(E)$; other cut-off choices are possible, and physical forms are the same: $\varphi(mb)$ falls off rapidly for increasing b.

Rescaling the $\int d[\chi]\, e^{\frac{i}{4}\int d^4w\, \chi^2(w)}$, for $\chi(w) \Rightarrow \chi(w^0)$,

$\int d^4w\, \chi^2(w) \to \Delta^2 \chi^2(w_0)$, where $\Delta^2 \sim$ size of 4-volume about w_0:
Choose: $\Delta^2 \sim \frac{1}{m_\pi^2 E p_L} \sim \frac{1}{m_\pi^2 E^2}$, $\Delta \sim \frac{1}{m_\pi E}$.

If $\Delta\chi = \bar{\chi}$, $\frac{i}{4}\int d^4w\, \chi^2(w) \to \frac{i}{4}\bar{\chi}^2$, $\int d[\chi] \to \prod_{a\,\mu\nu} \int d^4\chi^a_{\mu\nu}(w_0)$

$\to \prod_{a\,\mu\nu} \Delta^{-1} \cdot \int d^4 \bar{\chi}^a_{\mu\nu}$,

and: $(f\cdot\chi)^{-1} \to \Delta(f\cdot\bar{\chi})^{-1}$,

So that: $g\,\delta^{(2)}(\vec{b})(f\cdot\chi)^{-1} \to \bar{\varphi} g\, m^2 \Delta \sim g E^2 \frac{\bar{\varphi}}{m_\pi E} \sim g\left(\frac{E}{m_\pi}\right)\bar{\varphi}(bE)$,

$\bar{\varphi}(bE) \sim e^{-b^2E^2/4}$

and falls off rapidly for increasing b.
For $b \sim 0$, $g \gg 1$, $E/m_\pi \gg 1$,
$\sim g\left(\frac{E}{m_\pi}\right) \Omega^{I}_a(0)\, \Omega^{II}_b(0)\, (f\cdot\bar{\chi})^{-1} \Big|_{30}^{ab}$ is "large"

What this means, physically; depends on the size of this exp. factor, which generates changes in color structure.

- For large b, $g\varphi \to 0$, & no change in color (no $\lambda_{I,II}$ brought down, as $\int d[\Omega] \to \delta[\alpha]$, $(e^{\lambda\alpha}) \to 1$.
- for small b: $\lambda_{I,II}$ appear, and initial → final matrix elements ↓.
- ∴ Suppose we have a Bd. state, a πion, made from $Q + \bar{Q}$. At distances of $b \sim 1/m_\pi$, we'll have attraction due to effectively-coherent (multi gluon) exchange. The $Q + \bar{Q}$ ∴ approach, b ↓, and color fluctuations begin, so that effective attraction ↓,

and $Q + \bar{Q}$ move apart. Then, attraction starts again ... and the process repeats ∞ of times.

- Like an MIT Bag Model; or Effective Asympt. Freedom.
- Details certainly subject to change ... but QCD physics appears and is non-perturbative!
- Numerical integration, over $\int d^4\bar{x}$, $\int d^n\Omega_I(b) \int d^n\Omega_{II}(b)$ still needed for precise evaluation, for scattering and π - bound state.
- Presently working on definition of hadrons from QCD, and definition of hadron-hadron scatt. and production.
- All this due to Effective locality, as seen in simplified Eikonal Model. But EL can be shown to be rigorously true, for all QCD processes, without approximation, and without exception.