On the Collinear Behavior of Hot QCD

K. Bouakaz [a,b] and T. Grandou [b]

[a] Laboratoire de Physique des Particules et de Physique Statistique, Ecole Normale Supérieure, Algiers

[b] Institut Non Linéaire de Nice, France

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Outline

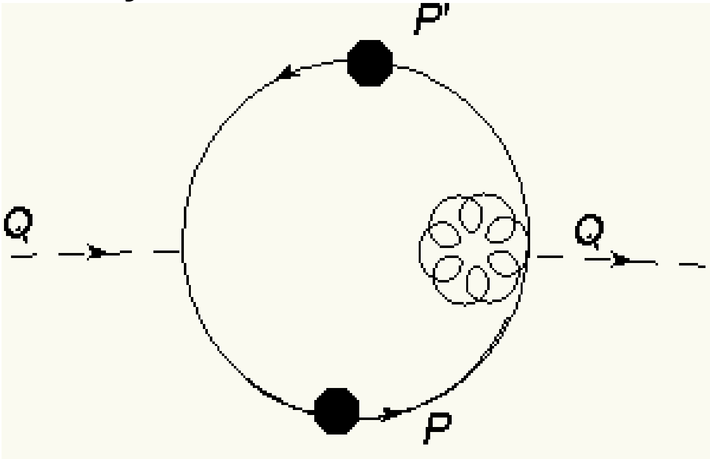
- Introduction
- Two effective vertex contributions
- Conclusion and Prospects

Introduction

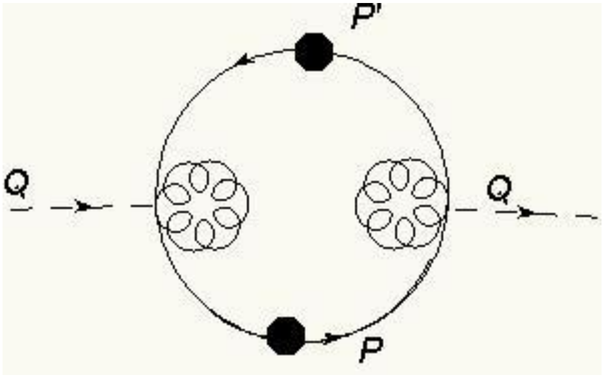
- Hadronic collisions $\rightarrow$ formation of QGP
- Several quantities of interest in a QCD plasma $\rightarrow$ damping rates
- At high temperature: The problem of infrared divergences $\rightarrow$ necessity of Resummation program
- First application of RP $\Rightarrow \gamma_t(0) = \frac{g^2 N_c T}{24\pi} a_{t0}$
- The problem of collinear singularity with RP in the calculation of the soft real photon emission rate out of a QGP in thermal equilibrium

• No problem with the use of correct sequence of angular average and discontinuity in $\Pi_R^{(*,*1)}(Q)$

In T. Grandou J. Math . Phys. Vol. 45, No 12, December 2004



• We calculate $\Pi_R^{(*,*2)}(Q)$ with the correct sequence



Two effective vertex contributions

- The soft real photon emission rate out of a Quark-Gluon Plasma in thermal equilibrium involves, in particular, the calculation of the quantity

$$\Pi_R(Q) = i\int \frac{d^4P}{(2\pi)^4}\left(1-2n_F(p_0)\right) disc_{p_0} Tr\left\{ {}^*S_R(P)\, {}^*\Gamma_\mu\left(P_R, Q_R, -P'_A\right) {}^*S_R(P')\, {}^*\Gamma^\mu\left(P_R, Q_R, -P'_A\right)\right\}$$

$${}^*S_R(P) = \frac{i}{\not P - \Sigma_\alpha(P) + i\varepsilon_\alpha p_0}, \quad \alpha = R, A, \qquad \varepsilon_R = -\varepsilon_A = \varepsilon$$

$$\Sigma_\alpha(P) = m^2 \int \frac{d\hat K}{4\pi} \frac{\hat{\not K}}{\hat K . P + i\varepsilon_\alpha}, \qquad m^2 = C_F \frac{g^2T^2}{8}$$

$${}^*\Gamma_\mu\left(P_\alpha, Q_\beta, P'_\delta\right) = -ie\left(\gamma_\mu + \Gamma_\mu^{HTL}\left(P_\alpha, Q_\beta, P'_\delta\right)\right)$$

$$\Gamma_\mu^{HTL}\left(P_\alpha, Q_\beta, P'_\delta\right) = m^2 \int \frac{d\hat K}{4\pi} \frac{\hat K_\mu \hat{\not K}}{\left(\hat K . P + i\varepsilon_\alpha\right)\left(\hat K . P' + i\varepsilon_\delta\right)} \quad \hat K = \left(1, \hat k\right)$$

- Using the usual prametresation of the efective fermionic propagator

$$ {}^{*}S_R(P)=\frac{i}{2}\sum_{s=\pm1}\hat{P}_s\ {}^{*}\Delta^s(p_0,p) $$

$$ \text{with}\quad \hat{P}_s=(1,s\hat{p})\ \text{and}\quad {}^{*}\Delta^s(p_0,p)=\left(p_0-sp-\frac{m^2}{2p}\left[\left(1-s\frac{p_0}{p}\right)\ln\frac{p_0+p}{p_0-p}+2s\right]\right)^{-1} $$

$$ \Pi_R^{(*,*2)}(Q)=+ie^2\int\frac{d^4P}{(2\pi)^4}(1-2n_F(p_0))\sum_{s,s'=\pm1}{}^{*}\Delta_R^{s'}(P')\,disc_{p_0}\left\{{}^{*}\Delta_R^{s}(P)\int\frac{d\hat{K}}{4\pi}\int\frac{d\hat{K}'}{4\pi}\hat{K}.\hat{K}'\right. $$

$$ \left.\frac{\hat{K}.\hat{P}_s\hat{K}'.\hat{P}'_{s'}+\hat{K}.\hat{P}'_{s'}\hat{K}'.\hat{P}_s-\hat{K}.\hat{K}'\hat{P}_s.\hat{P}'_{s'}}{(\hat{K}.\hat{P}+i\varepsilon)(\hat{K}.\hat{P}'+i\varepsilon)(\hat{K}'.\hat{P}+i\varepsilon)(\hat{K}'.\hat{P}'+i\varepsilon)}\right\} $$

Introducing the angular integral

$$ W(P,P')=\int\frac{d\hat{K}}{4\pi}\int\frac{d\hat{K}'}{4\pi}\hat{K}.\hat{K}'\frac{\hat{K}.\hat{P}_s\hat{K}'.\hat{P}'_{s'}+\hat{K}.\hat{P}'_{s'}\hat{K}'.\hat{P}_s-\hat{K}.\hat{K}'\hat{P}_s.\hat{P}'_{s'}}{(\hat{K}.\hat{P}+i\varepsilon)(\hat{K}.\hat{P}'+i\varepsilon)(\hat{K}'.\hat{P}+i\varepsilon)(\hat{K}'.\hat{P}'+i\varepsilon)} $$

We get

$$ \mathrm{Im}\,\Pi_R^{(*,*2)}(Q)=\pi e^2m^4\int\frac{d^4P}{(2\pi)^4}(1-2n_F(p_0))\sum_{s,s'=\pm1}\left\{-2\pi\beta_s(P)\beta_{s'}(P')W(P,P')\right. $$

$$ \left.+\left(\alpha_s(P)\beta_{s'}(P')+\alpha_{s'}(P')\beta_s(P)\right)\left(-i\,disc_{p_0}W(P,P')\right)\right\} $$

We have three contribution

$$\beta_s^{(p)}(p_0, p)\beta_{s'}^{(p)}(p'_0, p')$$

$$\beta_s^{(p)}(p_0, p)\beta_{s'}^{(c)}(p'_0, p')$$

$$\beta_s^{(c)}(p_0, p)\beta_{s'}^{(c)}(p'_0, p')$$

- The first contribution

$$-2\pi^2 e^2 m^4 \int \frac{d^4P}{(2\pi)^4}(1-2n_F(p_0)) \sum_{s,s'=\pm1} \left\{ \frac{\omega_s^2(p)-p^2}{2m^2}\delta(p_0-\omega_s(p)) + \frac{\omega_{-s}^2(p)-p^2}{2m^2}\delta(p_0+\omega_{-s}(p)) \right\}$$

$$\left\{ \frac{\omega_{s'}^2(p')-p'^2}{2m^2}\delta(p'_0-\omega_{s'}(p')) + \frac{\omega_{-s'}^2(p')-p'^2}{2m^2}\delta(p'_0+\omega_{-s'}(p')) \right\} \int_{\hat{K}}\int_{\hat{K}'} \hat{K}.\hat{K}' \frac{\hat{K}.\hat{P}_s \hat{K}'.\hat{P}'_{s'} + \hat{K}.\hat{P}'_{s'}\hat{K}'.\hat{P}_s - \hat{K}.\hat{K}'\hat{P}_s.\hat{P}'_{s'}}{(\hat{K}.\hat{P}+i\varepsilon)(\hat{K}.\hat{P}'+i\varepsilon)(\hat{K}'.\hat{P}+i\varepsilon)(\hat{K}'.\hat{P}'+i\varepsilon)}$$

For $s=\pm$ and all p we have $\omega_s(p) > p$, So the integral is regular

- The third case has been studied in JMP 2004 and shown to lead to singularity free contribution, The intermediate crossed case

$$-2\pi^2 e^2 m^4 \int_P \left(1-2n_F\left(p_0\right)\right) \sum_{s,s'=\pm1} \beta_s^{(p)}\left(p_0,p\right) \frac{\omega_{s'}^2\left(p'\right)-p'^2}{2m^2} \delta\left(p_0'-\omega_{s'}\left(p'\right)\right) \frac{ss'}{pp'}$$

$$\int_{\hat{K}}\int_{\hat{K}'} \hat{K}.\hat{K}' \frac{\hat{K}.\hat{P}_s\hat{K}'.\hat{P}_{s'}' + \hat{K}.\hat{P}_{s'}'\hat{K}'.\hat{P}_s - \hat{K}.\hat{K}'\hat{P}_s.\hat{P}_{s'}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}.\hat{P}'+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)}$$

The two first terms are symmetric in the exchange of $\hat{K}$ and $\hat{K}'$

The angular integral corresponding to the term $\hat{K}.\hat{P}_s\hat{K}'.\hat{P}_{s'}'$ reads

$$\int_{\hat{K}}\int_{\hat{K}'} \frac{\hat{K}.\hat{K}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)} \left\{ \frac{ss'}{pp'} + \frac{s'}{p'}\left(1-\frac{sp_0}{p}\right)\frac{1}{\hat{K}.\hat{P}+i\varepsilon} + \frac{s}{p}\left(1-\frac{s'p_0'}{p'}\right)\frac{1}{\hat{K}'.\hat{P}'+i\varepsilon} + \left(1-\frac{sp_0}{p}\right)\left(1-\frac{s'p_0'}{p'}\right)\frac{1}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)} \right\}$$

$$-2\pi^2 e^2 \int_P \left(1-2n_F\left(p_0\right)\right) \sum_{s,s'=\pm1} \frac{\omega_s^2\left(p\right)-p^2}{2m^2} \delta\left(p_0-\omega_s\left(p\right)\right) \int_{\hat{K}}\int_{\hat{K}'} \frac{\hat{K}.\hat{K}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)} \left\{ \frac{ss'}{pp'} + \frac{s'}{p'}\left(1-\frac{sp_0}{p}\right)\frac{1}{\hat{K}.\hat{P}+i\varepsilon} \right.$$

$$\left. + \frac{s}{p}\left(1-\frac{s'p_0'}{p'}\right)\frac{1}{\hat{K}'.\hat{P}'+i\varepsilon} + \left(1-\frac{sp_0}{p}\right)\left(1-\frac{s'p_0'}{p'}\right)\frac{1}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)} \right\}$$

The angular integral corresponding to the term ss'/pp' contributes

$$-2\pi^2 e^2 \int_P \left(1-2n_F\left(p_0\right)\right) \sum_{s,s'=\pm 1} \beta_s^{(p)}\left(p_0, p\right) \frac{\omega_{s'}^2\left(p'\right)-p'^2}{2m^2} \delta\left(p_0' - \omega_{s'}\left(p'\right)\right) \frac{ss'}{pp'} \Sigma_R\left(P\right) . \Sigma_R\left(P'\right)$$

where the "self energy four-vector" has components

$$\Sigma_\alpha^0\left(P\right) = \frac{m^2}{2p} \ln \frac{p_0 + p}{p_0 - p},$$

$$\Sigma_\alpha^i\left(P\right) = \left(\frac{p^i}{p} \equiv \hat{p}^i\right) \frac{m^2}{p} Q_1\left(\frac{p_0}{p}\right)$$

These components lead to the obviously free contributions of

$$-2\pi^2 e^2 \int \frac{d^3 p}{\left(2\pi\right)^3} \int_{-p}^{+p} \frac{dp_0}{2\pi} \left(1-2n_F\left(p_0\right)\right) \sum_{s,s'=\pm 1} \beta_s^{(p)}\left(p_0, p\right) \frac{\omega_{s'}^2\left(p'\right)-p'^2}{2m^2} \delta\left(p_0' - \omega_{s'}\left(p'\right)\right) \frac{ss'}{pp'}$$

$$\Sigma_R\left(\omega_{s'}\left(p'\right), p'\right) . \frac{m^2}{2p} \left(\ln \frac{p_0 + p}{p_0 - p}, \hat{p}^i \left[\frac{p_0}{p} \ln \frac{p_0 + p}{p_0 - p} - 2 \right] \right)$$

The second term

$$\frac{s'}{p'}\left(1-\frac{sp_0}{p}\right)\int_{\hat{K}}\int_{\hat{K}'}\frac{\hat{K}.\hat{K}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)}\frac{1}{\hat{K}.\hat{P}+i\varepsilon}=\frac{s'}{p'}\left(1-\frac{sp_0}{p}\right)\left\{\frac{1}{p^2}Q_1\left(\frac{p_0}{p}\right)\frac{1}{2p'}\ln\frac{p_0'+p'}{p_0'-p'}\right.$$

$$\left.+\frac{1}{p}\left(\frac{p_0}{p}-\frac{P^2}{p^2}Q_0\left(\frac{p_0}{p}\right)\right)\frac{1}{2Q.P}\ln\frac{P'^2}{P^2}\right\}$$

$$-2\pi^2e^2m^4\int_P\left(1-2n_F\left(p_0\right)\right)\sum_{s,s'=\pm1}\beta_s^{(p)}\left(p_0,p\right)\frac{\omega_{s'}^2\left(p'\right)-p'^2}{2m^2}\delta\left(p_0'-\omega_{s'}\left(p'\right)\right)\frac{s}{p}\left(1-\frac{s'p_0'}{p'}\right)\left\{\frac{1}{p'^2}Q_1\left(\frac{\omega_{s'}\left(p'\right)}{p'}\right)\frac{1}{2p}\ln\frac{p_0+p}{p_0-p}\right.$$

$$\left.+\frac{1}{p}\left(\frac{\omega_{s'}\left(p'\right)}{p}-\frac{P'^2}{p'^2}Q_0\left(\frac{p_0}{p}\right)\right)\frac{1}{2q\left(\omega_{s'}\left(p'\right)-p'y\right)}\ln\frac{\omega_{s'}^2\left(p'\right)-p'^2}{P^2}\right\}$$

We have used $\quad 2Q.P=2Q.P'=2q\left(\omega_{s'}\left(p'\right)-p'y\right)>0 \quad$ and $-p\le p_0\le +p$

$$-2\pi^2e^2m^4\int\frac{d^3p}{(2\pi)^3}\int_{-p}^{+p}\frac{dp_0}{2\pi}\left(1-2n_F\left(p_0\right)\right)\sum_{s,s'=\pm1}\beta_s^{(p)}\left(p_0,p\right)\frac{\omega_{s'}^2\left(p'\right)-p'^2}{2m^2}\delta\left(p_0'-\omega_{s'}\left(p'\right)\right)\frac{s'}{p'}\left(1-\frac{sp_0}{p}\right)$$

$$\left\{\frac{1}{p^2}Q_1\left(\frac{p_0}{p}\right)\frac{1}{2p'}\ln\frac{\omega_{s'}\left(p'\right)+p}{\omega_{s'}\left(p'\right)-p'}+\frac{1}{p}\left(\frac{p_0}{p}-\frac{P^2}{p^2}Q_0\left(\frac{p_0}{p}\right)\right)\frac{1}{2Q.P}\ln\frac{\omega_{s'}^2\left(p'\right)-p'^2}{P^2}\right\}$$

Regular contribution

The same behavior for the third term $\left(1-\frac{sp_0}{p}\right)\left(1-\frac{s'p'_0}{p'}\right)\int_{\hat{K}}\int_{\hat{K}'}\frac{\hat{K}.\hat{K}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}.\hat{P}'+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)}$

$$\int\frac{d\hat{K}}{4\pi}\frac{\hat{K}^{\mu}}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}.\hat{P}'+i\varepsilon\right)}\int\frac{d\hat{K}'}{4\pi}\frac{\hat{K}^{\mu}}{\left(\hat{K}'.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)}=\sum_{i,j=0}^{3}\left(\sum_{k=-4}^{+1}a_{ij}^{k}Z^{k}\right)F_iF_j$$

$$a_{00}^{0}=1,\quad a_{11}^{-2}=-q^4$$

$$a_{12}^{-3}=2q^4P^2,\qquad a_{12}^{-2}=-q^3\left(p_0+px\right),\qquad a_{12}^{-1}=-\frac{q^2}{2}$$

$$a_{13}^{-3}=-2q^4\left(P^2\right)^2,\qquad a_{13}^{-2}=2q^3P^2\left(p_0+px\right),\ a_{13}^{-1}=-2q^2p_0px$$

$$a_{23}^{-4}=2q^4\left(P^2\right)^3,\qquad a_{23}^{-3}=-3q^3\left(P^2\right)^2\left(p_0+px\right),\ a_{23}^{0}=-p_0px$$

$$a_{23}^{-2}=2q^2P^2p\left(p+xp_0\right)+q^2P^2p_0\left(p_0+xp\right)-\frac{q^2\left(P^2\right)^2}{2},\qquad a_{23}^{-1}=qP^2px+\frac{1}{2}qP^2p_0-2qp^2p_0$$

$$a_{22}^{-4}=-q^4\left(P^2\right)^2,\ a_{22}^{-3}=q^3P^2\left(p_0+xp\right),\quad a_{22}^{-2}=-q^2p^2+\frac{q^2P^2}{2},\quad a_{22}^{-1}=-\frac{q}{2}\left(p_0+px\right)$$

$$a_{33}^{-4}=-q^4\left(P^2\right)^4,\qquad a_{33}^{-3}=q^3\left(P^2\right)^3\left(2p_0+2px\right),\ a_{33}^{-2}=-q^2\left(P^2\right)^2\left(p_0^2+p^2+4pp_0x\right)$$

$$a_{33}^{-1}=2qpp_0P^2\left(p+xp_0\right),\ a_{33}^{0}=-p^2p_0^2$$

$$\int_0^1 \frac{ds}{R^2(s)} = \frac{1}{Z}\ln\frac{P'^2}{P^2} \equiv F_0(P,Q)$$

$$\int_0^1 \frac{ds}{r^2(s)} = \frac{1}{qp\sqrt{1-x^2}}\arctan\frac{q\sqrt{1-x^2}}{p+qx} \equiv F_2(P,Q)$$

$$\int_0^1 ds\frac{s}{r^2(s)} = \frac{1}{2q^2}\ln\frac{p'^2}{p^2} - \frac{px}{q}F_2 \equiv F_1 - \frac{px}{q}F_2$$

$$\int_0^1 ds\frac{s}{R^2(s)r^2(s)} = \frac{1}{\left(P^2\vec{p}' - \vec{p}P'^2\right)^2}\left\{\left(q^2P^2 - 2pqx2Q.P\right)F_2 + 2Q.P\ln\frac{P'^2}{P^2} - 2Q.P\ln\frac{p'(x)}{p}\right\} \equiv F_3(P,Q)$$

$$R^2(s) = P^2 + 2Q.Ps$$

$$r^2(s) = p^2 + 2qpxs + q^2s^2$$

The last contribution to crossed case

$$-2\pi^2 e^2 m^4 \int \frac{d^4P}{(2\pi)^4}\left(1-2n_F(p_0)\right)\sum_{s,s'=\pm1}\left\{\frac{\omega_s^2(p)-p^2}{2m^2}\delta\left(p_0-\omega_s(p)\right)+\frac{\omega_{-s}^2(p)-p^2}{2m^2}\delta\left(p_0+\omega_{-s}(p)\right)\right\}$$

$$\left\{\frac{\omega_{s'}^2(p')-p'^2}{2m^2}\delta\left(p_0'-\omega_{s'}(p')\right)+\frac{\omega_{-s'}^2(p')-p'^2}{2m^2}\delta\left(p_0'+\omega_{-s'}(p')\right)\right\}\int_{\hat{K}}\int_{\hat{K}'}\hat{K}.\hat{K}'\frac{\hat{K}.\hat{P}_s\hat{K}'.\hat{P}_{s'}'+\hat{K}.\hat{P}_{s'}'\hat{K}'.\hat{P}_s-\hat{K}.\hat{K}'\hat{P}_s.\hat{P}_{s'}'}{\left(\hat{K}.\hat{P}+i\varepsilon\right)\left(\hat{K}.\hat{P}'+i\varepsilon\right)\left(\hat{K}'.\hat{P}+i\varepsilon\right)\left(\hat{K}'.\hat{P}'+i\varepsilon\right)}$$

$$W_2(P,P')=\int_0^1 ds\int_0^1 ds'\int\frac{d\hat{K}}{4\pi}\int\frac{d\hat{K}'}{4\pi}\frac{\left(\hat{K}.\hat{K}'\right)^2}{\left(\hat{K}.R(s)+i\varepsilon\right)^2\left(\hat{K}'.R(s')+i\varepsilon\right)^2}$$

$$W_2(P,P')=2W_1(P,P')-\frac{1}{Z^2}\ln^2\frac{P'^2}{P^2}+\int_0^1 ds\int_0^1 ds'\int\frac{d\hat{K}}{4\pi}\int\frac{d\hat{K}'}{4\pi}\frac{\hat{K}^i.\hat{K}^j.\hat{K}_i'.\hat{K}_j'}{\left(\hat{K}.R(s)+i\varepsilon\right)^2\left(\hat{K}'.R(s')+i\varepsilon\right)^2}$$

$$\int\frac{d\hat{K}}{4\pi}\frac{\hat{K}^i.\hat{K}^j}{\left(\hat{K}.R(s)+i\varepsilon\right)^2}=-\frac{g^{ij}}{r^2}Q_1\left(\frac{r_0}{r}\right)-\frac{r^ir^j}{r^2}\left(\frac{3}{r^2}Q_1\left(\frac{r_0}{r}\right)-\frac{1}{R^2(s)+i\varepsilon r_0}\right)$$

$$\int\frac{d\hat{K}'}{4\pi}\frac{\hat{K}^i}{\left(\hat{K}.R(s)+i\varepsilon\right)^2}=\frac{r^i}{r^2}\left(\frac{1}{2r}\ln\frac{r_0+r}{r_0-r}-\frac{r_0}{R^2(s)+i\varepsilon r_0}\right)$$

$$\int_0^1 ds\int_0^1 ds'\int \frac{d\hat{K}}{4\pi}\int \frac{d\hat{K}'}{4\pi}\frac{\hat{K}^i.\hat{K}^j.\hat{K}'_i.\hat{K}'_j}{\left(\hat{K}.R(s)+i\varepsilon\right)^2\left(\hat{K}'.R(s')+i\varepsilon\right)^2} = -3\left(\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)}\right)^2$$

$$+2\left(\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)}\right)\left(\int_0^1 \frac{ds'}{R^2(s')+i\varepsilon r_0(s')}\right)+9\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)}\int_0^1 ds'\left[\hat{r}(s).\hat{r}(s')\right]^2\frac{Q_1(R(s'))}{r^2(s')}$$

$$-6\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)}\int_0^1 ds'\,\frac{\left[\hat{r}(s).\hat{r}(s')\right]^2}{R^2(s')+i\varepsilon r_0(s')}+\int_0^1 \frac{ds}{R^2(s)+i\varepsilon r_0(s)}\int_0^1 ds'\,\frac{\left[\hat{r}(s).\hat{r}(s')\right]^2}{R^2(s')+i\varepsilon r_0(s')}$$

$$2F_0\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)} = \frac{F_0}{qp\left(1-x^2\right)}\left(\left(p_0x-p+\frac{Z}{2p}\right)\frac{\ln X'}{p'}-\left(p_0x-p\right)\frac{\ln X}{p}\right)-\frac{\left(ZF_0\right)^2}{2q^2p^2\left(1-x^2\right)}$$

$$-3\left(\int_0^1 ds\,\frac{Q_1(R(s))}{r^2(s)}\right)^2 = \frac{-3}{4q^2p^2\left(1-x^2\right)^2}\left(-\frac{Z^2F_0}{2qp}+\left(p_0x-p+\frac{Z}{2p}\right)\frac{\ln X'}{p'}-\left(p_0x-p\right)\frac{\ln X}{p}\right)^2$$

The fifth term

$$\int_0^1 \frac{ds}{R^2(s)+i\varepsilon r_0(s)} \int_0^1 ds' \frac{\left[\hat{r}(s).\hat{r}(s')\right]^2}{R^2(s')+i\varepsilon r_0(s')} = \sum_{i,j=1} \left(\sum_{k=-2}^{+1} a_{ij}^k Z^k \right) F_i F_j$$

$$a_{00}^0 = 1$$

$$a_{22}^{-2} = 2q^2 p^2 \left(1 - x^2\right)$$

$$a_{33}^{-2} = 2q^2 p^2 \left(1 - x^2\right)\left(P^2\right)^2, \qquad a_{33}^{-1} = -4qp^3 x \left(1 - x^2\right) P^2, \quad a_{33}^0 = 2p^4 \left(1 - x^2\right)$$

$$a_{13}^0 = -2p^2 \left(1 - x^2\right)$$

$$a_{23}^{-2} = -4q^2 p^2 \left(1 - x^2\right) P^2, \quad a_{23}^{-1} = 4qp^3 x \left(1 - x^2\right)$$

The fourth term

$$-6F_0\left\{\frac{-1}{2}F_2+\frac{1}{2q}\left(\frac{q+px}{p'^2}-\frac{x}{p}\right)+\frac{p_0}{2}I_3+\frac{q}{2}I_3'-\frac{p_0p^2\left(1-x^2\right)}{2}I_5-\frac{qp^2\left(1-x^2\right)}{2}I_5'\right\}$$

$$-6p^2\left(1-x^2\right)F_3\left\{\frac{-1}{p'^2}+\left(px-\frac{p_0}{2}\right)I_3-\frac{q}{2}I_3'+\frac{p^2}{2q}ZI_5+qp\left(p\left(1-x^2\right)+\frac{x}{2q}Z\right)I_5'\right\}$$

$$-12qp^2\left(1-x^2\right)\frac{F_2-P^2F_3}{Z}\left\{\frac{1}{2qp}\left(\frac{p}{p'^2}-\frac{1}{p}\right)+\frac{1}{2}I_3-\frac{q}{2}I_3'+\frac{p\left(p_0x-p\right)}{2}I_5+\frac{Z}{4}I_5'\right\}$$

The third term

$$\frac{9}{2}\left(p_0 I_3 + qI_3' - 2F_2\right)\left\{\frac{-F_2}{2} - \frac{x(q+px)}{2pp'^2} + \frac{1-x^2}{2p'^2} + \frac{p_0}{2}I_3 + \frac{q}{2}I_3' - \frac{p_0 p^2\left(1-x^2\right)}{2}I_5 - \frac{p_0 p^2\left(1-x^2\right)}{2}I_5'\right\}$$

$$+\frac{9}{2}\left(p^2\left(1-x^2\right)\left(p_0 I_5 + qI_5'\right) + \frac{x(q+px)}{pp'^2} - \frac{1-x^2}{p'^2} - F_2\right)\left\{-\frac{1}{2p'^2} + \left(px - \frac{p_0}{2}\right)I_3 - \frac{q}{2}I_3' + \frac{p^2 Z}{2q}I_5 + qp\left(p\left(1-x^2\right) + \frac{xZ}{2q}\right)I_5'\right\}$$

$$+\frac{9}{2}\left(p^2\left(1-x^2\right)\left(I_3 - p^2 I_5 + \left(\frac{Z}{2} - qpx\right)I_5'\right) - \frac{q+px}{p'^2} + pxF_2\right)\left\{\frac{1}{qp}\left(\frac{p}{p'^2} - \frac{1}{p}\right) + I_3 + p\left(p_0 x - p\right)I_5' + \frac{Z}{2}I_5'\right\}$$

$$I_3=\int_0^1 ds\,\frac{\ln X}{r^3(s)}=\frac{1}{qp^2\left(1-x^2\right)}\left(\left((q+px)\ln X'-px\ln X\right)\right.$$

$$\left.-\frac{ZF_0}{qp^2\left(1-x^2\right)}+2p_0F_3+2q\,\frac{F_2-P^2F_3}{Z}\right.$$

$$I_3'=\int_0^1 ds\,\frac{s\ln X}{r^3(s)}=\frac{1}{pq^2\left(1-x^2\right)}\left(p\,\frac{\ln X}{p}-(p+qx)\frac{\ln X'}{p'}\right)$$

$$+\frac{x}{pq^2\left(1-x^2\right)}ZF_0-2\,\frac{p^2}{q}F_3+2\left(p_0-2px\right)\frac{F_2-P^2F_3}{Z}$$

$$I_5=\frac{1}{3qp^2\left(1-x^2\right)}\left[\frac{q+px}{p'^3}\ln X'-\frac{x}{p^2}\ln X\right]+\frac{2}{3qp^4\left(1-x^2\right)^2}\left(\frac{q+px}{p'}\ln X'-x\ln X\right)$$

$$-\frac{2Z}{3qp^4\left(1-x^2\right)^2}F_0+\frac{2\left(p_0+px\right)}{3p^2\left(1-x^2\right)}F_3+\frac{4q}{3p^2\left(1-x^2\right)}\frac{F_2-P^2F_3}{Z}$$

$$+\frac{2q}{3Z}\left(\int_0^1\frac{ds}{r^4(s)}+\left(p_0^2+p^2-2p_0px\right)\int_0^1\frac{ds}{R^2(s)r^4(s)}\right)$$

$$I_5'=\frac{-1}{3q^2p\left(1-x^2\right)}\left[\frac{p+qx}{p'^3}\ln X'-\frac{1}{p^2}\ln X\right]-\frac{2x}{3q^2p^3\left(1-x^2\right)^2}\left(\frac{q+px}{p'}\ln X'-x\ln X\right)$$

$$+\frac{4x}{3qp^3\left(1-x^2\right)^2}\frac{Z}{2q}F_0-\frac{2x\left(p_0+px\right)}{3qp\left(1-x^2\right)}F_3-\frac{4x}{3p\left(1-x^2\right)}\frac{F_2-P^2F_3}{Z}$$

$$+\left(\frac{1}{3q}-\frac{2px}{3Z}\right)\left(\int_0^1\frac{ds}{r^4(s)}-\frac{2p_0\left(p_0^2+p^2-2p_0px\right)}{3Z}\int_0^1\frac{ds}{R^2(s)r^4(s)}\right)$$

We have $2Q.P=2Q.P'=2q\left(\omega_{s'}\left(p'\right)-p'y\right)>0$ and $-p\le p_0\le +p$ $\quad x<\frac{p_0}{p}$

The study of the behavior of diffrents contributions near $x=-1$

Conclusion and Prospects

- The proper sequence of angular integration and discontinuity operations lead to regular result
- Calculations along the correct sequence are ``an order of magnitude'' more difficult to carry out ,So We call for Nonperturbative calculation